Online Dictionary Learning for Sparse Coding

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What this talk is about

- Learning efficiently dictionaries (basis set) for sparse coding.
- Solving a large-scale matrix factorization problem.
- Making some large-scale image processing problems tractable.
- Proposing an algorithm which extends to NMF, sparse PCA,...

1. The Dictionary Learning Problem
2. Online Dictionary Learning
3. Extensions

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The Dictionary Learning Problem

$$\underbrace{\mathbf{y}}_{\text{measurements}} = \underbrace{\mathbf{x}_{orig}}_{\text{original image}} + \underbrace{\mathbf{w}}_{\text{noise}}$$

The Dictionary Learning Problem
[Elad & Aharon ('06)]

Solving the denoising problem

- Extract all overlapping 8×8 patches $\mathbf{x}_i$.
- Solve a matrix factorization problem:

$$\min_{\boldsymbol{\alpha}_i, \mathbf{D} \in \mathcal{C}} \sum_{i=1}^{n} \underbrace{\frac{1}{2}||\mathbf{x}_i - \mathbf{D}\boldsymbol{\alpha}_i||_2^2}_{\text{reconstruction}} + \underbrace{\lambda||\boldsymbol{\alpha}_i||_1}_{\text{sparsity}},$$

with $n > 100,000$

- Average the reconstruction of each patch.

The Dictionary Learning Problem
[Mairal, Bach, Ponce, Sapiro & Zisserman ('09)]

Denoising result

The Dictionary Learning Problem
[Mairal, Sapiro & Elad ('08)]

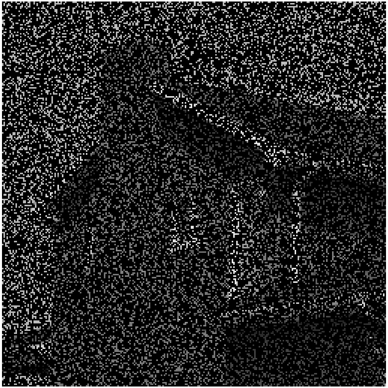
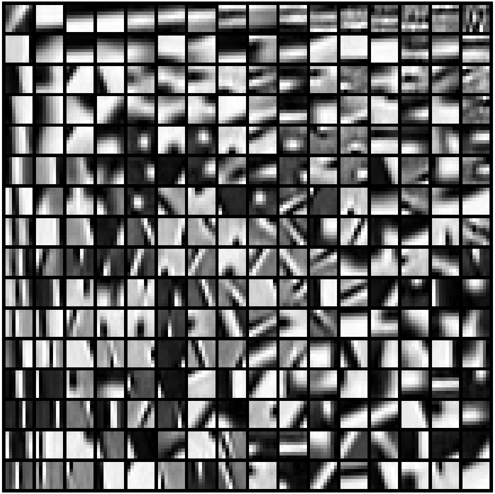


Image completion example

The Dictionary Learning Problem

What does **D** look like?



The Dictionary Learning Problem

$$\min_{\substack{\boldsymbol{\alpha} \in \mathbb{R}^{k \times n} \\ \mathbf{D} \in \mathcal{C}}} \sum_{i=1}^{n} \frac{1}{2} ||\mathbf{x}_i - \mathbf{D}\boldsymbol{\alpha}_i||_2^2 + \lambda ||\boldsymbol{\alpha}_i||_1$$

$$\mathcal{C} \triangleq \{\mathbf{D} \in \mathbb{R}^{m \times k} \;\; \text{s.t.} \;\; \forall j = 1, \ldots, k, \;\; ||\mathbf{d}_j||_2 \leq 1\}.$$

- Classical optimization alternates between $\mathbf{D}$ and $\boldsymbol{\alpha}$.
- Good results, but very slow!

1 The Dictionary Learning Problem

2 Online Dictionary Learning

3 Extensions

Online Dictionary Learning

Classical formulation of dictionary learning

$$\min_{\mathbf{D}\in\mathcal{C}} f_n(\mathbf{D}) = \min_{\mathbf{D}\in\mathcal{C}} \frac{1}{n}\sum_{i=1}^{n} l(\mathbf{x}_i, \mathbf{D}),$$

where

$$l(\mathbf{x}, \mathbf{D}) \triangleq \min_{\boldsymbol{\alpha}\in\mathbb{R}^k} \frac{1}{2}||\mathbf{x} - \mathbf{D}\boldsymbol{\alpha}||_2^2 + \lambda||\boldsymbol{\alpha}||_1.$$

Online Dictionary Learning

Which formulation are we interested in?

$$\min_{\mathbf{D}\in\mathcal{C}} \left[f(\mathbf{D}) \triangleq \mathbb{E}_x[l(\mathbf{x},\mathbf{D})] \approx \lim_{n\to+\infty} \frac{1}{n}\sum_{i=1}^{n} l(\mathbf{x}_i,\mathbf{D}) \right]$$

Online Dictionary Learning

Online learning can

- handle potentially infinite datasets,
- adapt to dynamic training sets,
- be dramatically faster than batch algorithms [Bottou & Bousquet ('08)].

Online Dictionary Learning

Proposed approach

for t=1,...,T **do**

  Draw $\mathbf{x}_t$

  Sparse Coding

$$\boldsymbol{\alpha}_t \leftarrow \underset{\boldsymbol{\alpha}\in\mathbb{R}^k}{\arg\min} \frac{1}{2}||\mathbf{x}_t - \mathbf{D}_{t-1}\boldsymbol{\alpha}||_2^2 + \lambda||\boldsymbol{\alpha}||_1,$$

  Dictionary Learning

$$\mathbf{D}_t \leftarrow \underset{\mathbf{D}\in\mathcal{C}}{\arg\min} \frac{1}{t}\sum_{i=1}^{t}\left(\frac{1}{2}||\mathbf{x}_i - \mathbf{D}\boldsymbol{\alpha}_i||_2^2 + \lambda||\boldsymbol{\alpha}_i||_1\right),$$

end for

Implementation details

- Use LARS for the sparse coding step,
- Use a block-coordinate approach for the dictionary update, with warm restart,
- Use a mini-batch.

Which guarantees do we have?

Under a few reasonable assumptions,

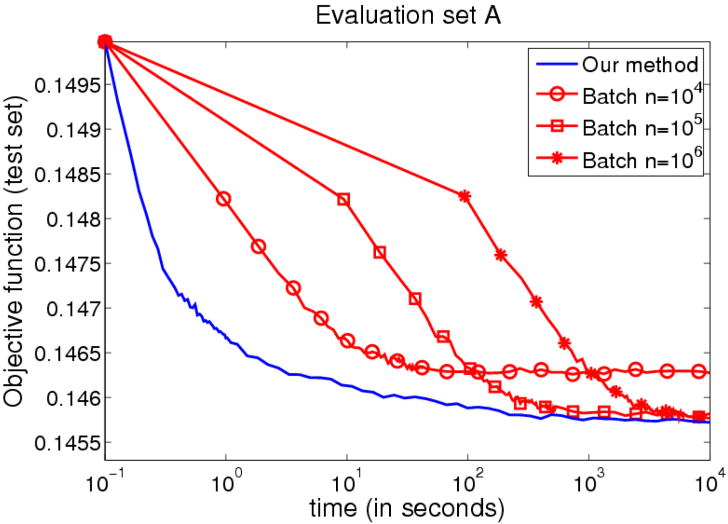
- we build a surrogate function $\hat{f}_t$ of the expected cost f verifying

$$\lim_{t \to +\infty} \hat{f}_t(\mathbf{D}_t) - f(\mathbf{D}_t) = 0,$$

- $\mathbf{D}_t$ is asymptotically close to a stationary point.

Online Dictionary Learning

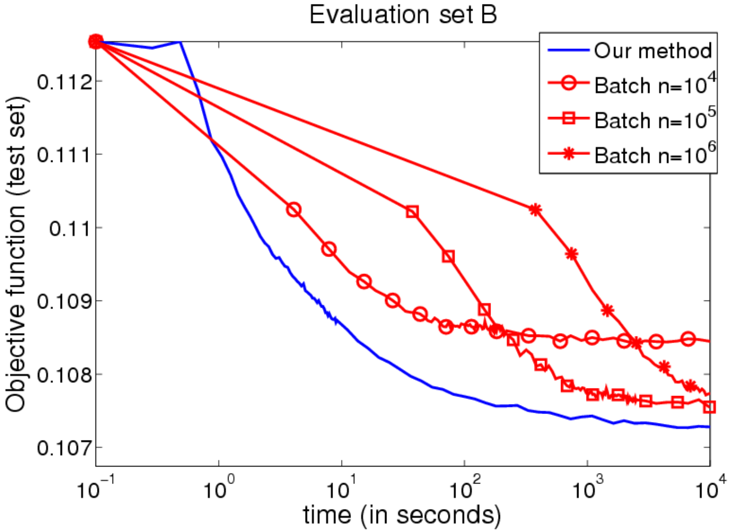
Experimental results, batch vs online



$m = 8 \times 8,\ k = 256$

Online Dictionary Learning

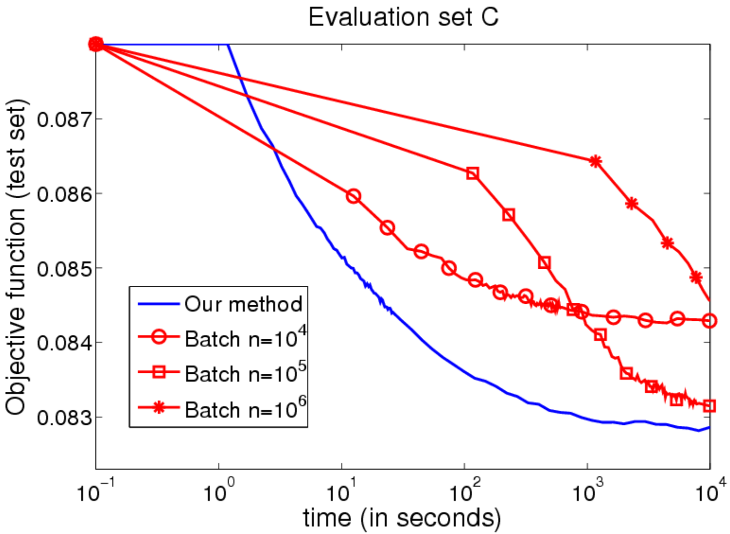
Experimental results, batch vs online



$$m = 12 \times 12 \times 3, \; k = 512$$

Online Dictionary Learning

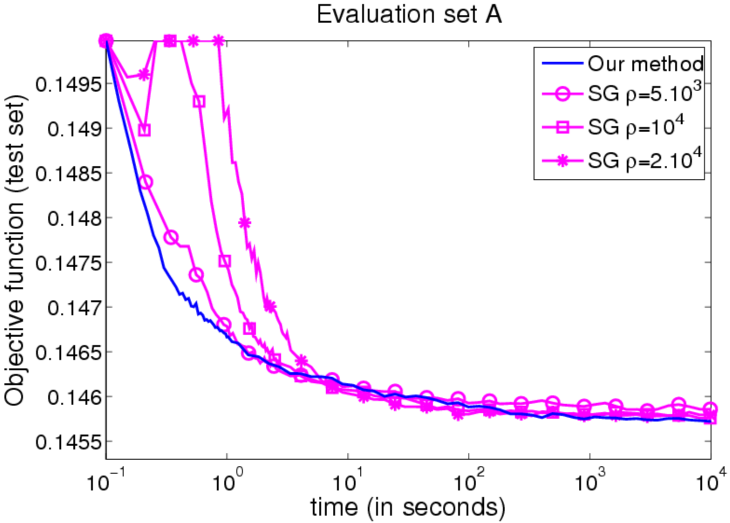
Experimental results, batch vs online



$$m = 16 \times 16,\ k = 1024$$

Online Dictionary Learning

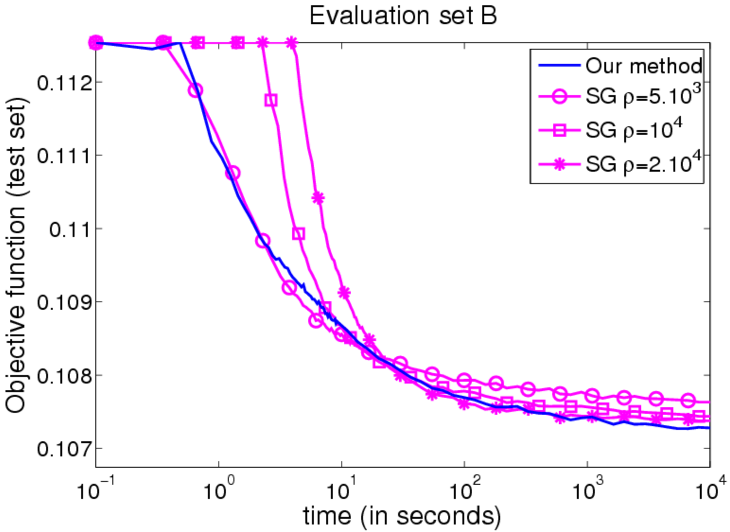
Experimental results, ODL vs SGD



$m = 8 \times 8, \ k = 256$

Online Dictionary Learning

Experimental results, ODL vs SGD



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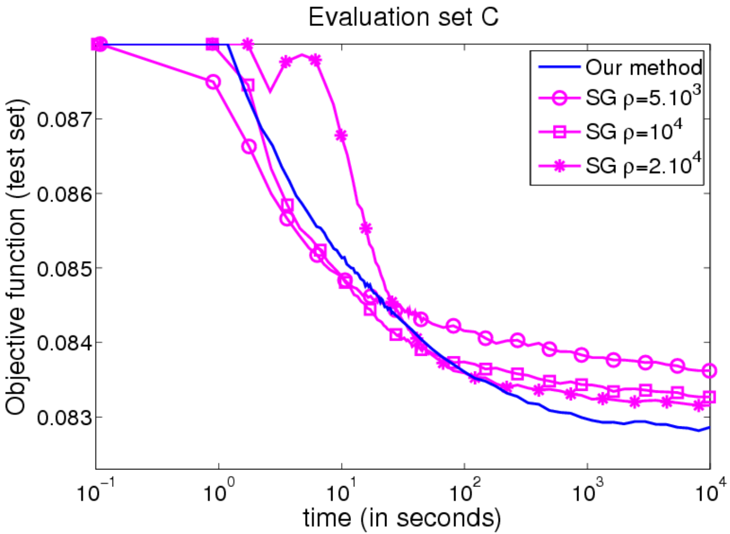
Online Dictionary Learning

Experimental results, ODL vs SGD



$$m = 16 \times 16,\ k = 1024$$

Online Dictionary Learning

Inpainting a 12-Mpixel photograph

Online Dictionary Learning

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Online Dictionary Learning
Inpainting a 12-Mpixel photograph

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Extension to NMF and sparse PCA

NMF extension

$$\min_{\substack{\boldsymbol{\alpha} \in \mathbb{R}^{k \times n} \\ \mathbf{D} \in \mathcal{C}}} \sum_{i=1}^{n} \frac{1}{2} ||\mathbf{x}_i - \mathbf{D}\boldsymbol{\alpha}_i||_2^2 \ \text{ s.t. } \ \boldsymbol{\alpha}_i \geq 0, \quad \mathbf{D} \geq 0.$$

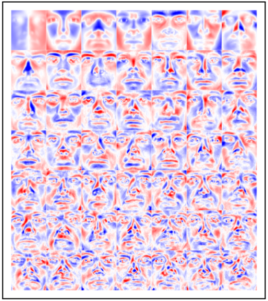
SPCA extension

$$\min_{\substack{\boldsymbol{\alpha} \in \mathbb{R}^{k \times n} \\ \mathbf{D} \in \mathcal{C}'}} \sum_{i=1}^{n} \frac{1}{2} ||\mathbf{x}_i - \mathbf{D}\boldsymbol{\alpha}_i||_2^2 + \lambda ||\boldsymbol{\alpha}_1||_1$$

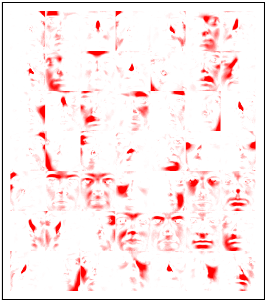
$$\mathcal{C}' \triangleq \{\mathbf{D} \in \mathbb{R}^{m \times k} \ \text{ s.t. } \ \forall j \ \ ||\mathbf{d}_j||_2^2 + \gamma ||\mathbf{d}_j||_1 \leq 1\}.$$

Extension to NMF and sparse PCA

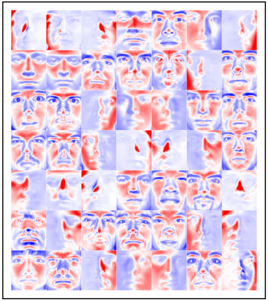
Faces: Extended Yale Database B



(a) PCA



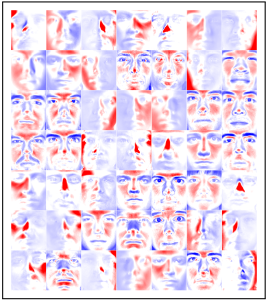
(b) NNMF



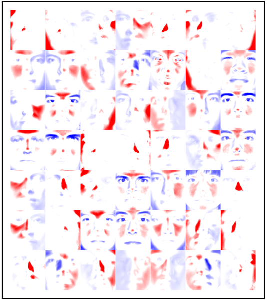
(c) DL

Extension to NMF and sparse PCA

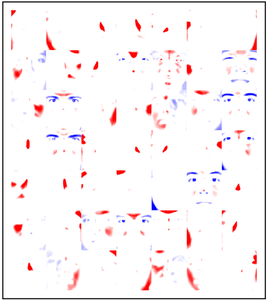
Faces: Extended Yale Database B



(d) SPCA, $\tau = 70\%$



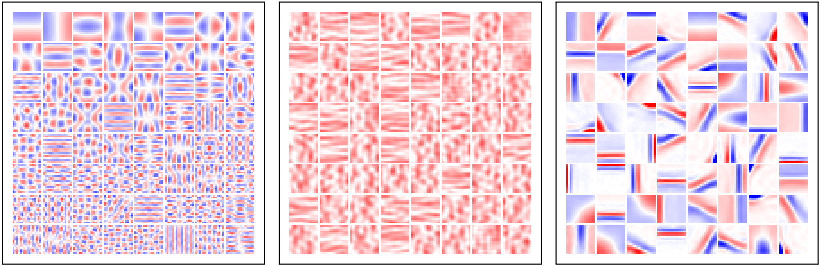
(e) SPCA, $\tau = 30\%$



(f) SPCA, $\tau = 10\%$

Extension to NMF and sparse PCA

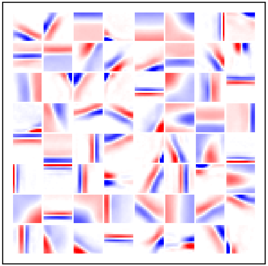
Natural Patches



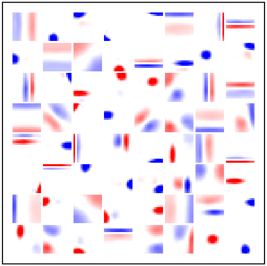
(a) PCA (b) NNMF (c) DL

Extension to NMF and sparse PCA

Natural Patches



(d) SPCA, $\tau = 70\%$



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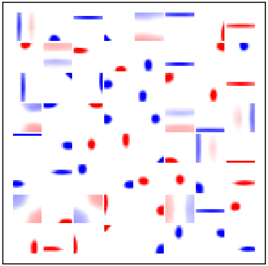

(f) SPCA, $\tau = 10\%$

Conclusion

Take-home message

- Online techniques are adapted to the dictionary learning problem.
- Our method makes some large-scale image processing tasks tractable—. . .
- . . . — and extends to various matrix factorization problems.