# Fisher kernels with application to image representation

Advanced Learning Models 2015-2016

Jakob Verbeek, January 21, 2016

Course website:

http://lear.inrialpes.fr/people/mairal/teaching/2015-2016/MSIAM

# Outline

- Homework: hand-in today January 21st
  - Subtraction of 0.5 points per day too late
- Data Challenge: available next week January 28th
  - Register with UJF/ENSIMAG email

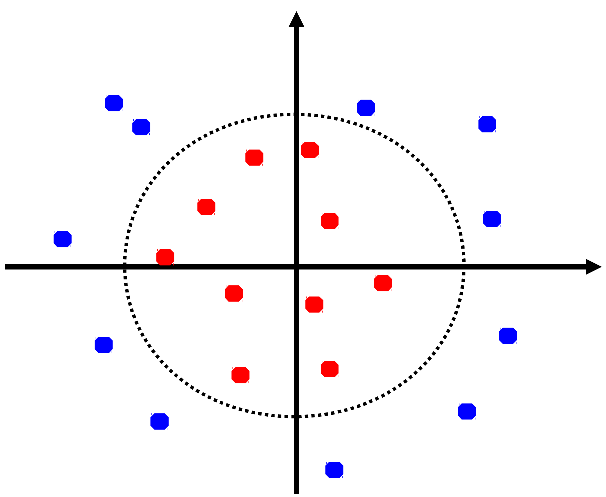
- This week
  - Fisher kernels, application to visual recognition with local features
  - Feed-forward neural networks
  - Convolutional neural networks (CNN)
  - Applications of CNN

- Next week: sequential data
  - Hidden Markov models
  - Recurrent neural networks
  - Applications of RNN

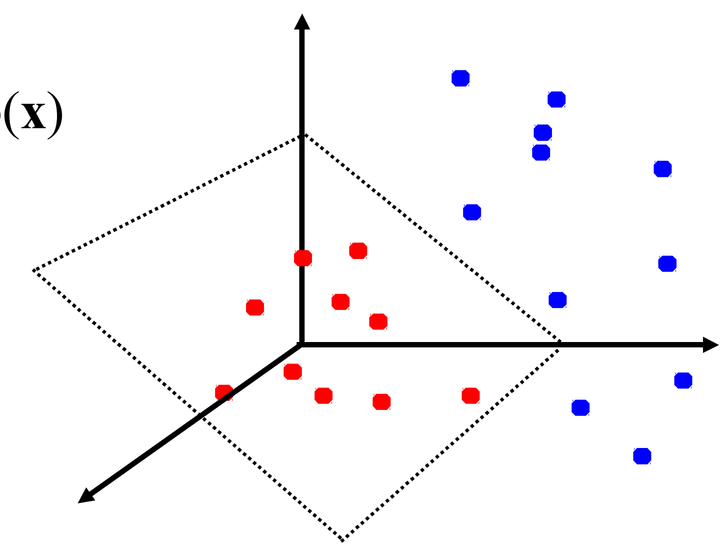
# A brief recap on kernel methods

- A way to achieve non-linear classification (or other data analysis) by using a kernel that computes inner products of data after non-linear transformation
  - Given the transformation, we can derive the kernel function.
- Conversely, if a kernel is positive definite, it is known to compute a dot-product in a (not necessarily finite dimensional) feature space
  - Given the kernel, we can determine the feature mapping function.

$$k(x_1, x_2) = \langle \varphi(x_1), \varphi(x_2) \rangle$$



$$\Phi: \ \mathbf{x} \longrightarrow \varphi(\mathbf{x})$$



# A brief recap on kernel methods

- Most often we start with data in a vector space, and map it to another feature space to allow for non-linear classification in the original space, using linear classification in the feature space
- Kernels can also be used to represent non-vectorial data, and to make them amenable to linear classification (or other linear data analysis) techniques
- For example, suppose we want to classify sets of points in a vector space, where the size of each set may vary

$$X=\{x_1, x_2, \ldots, x_N\} \quad \text{with} \quad x_i \in R^d$$

- We can define a representation of sets by concatenating the mean and variance of the set in each dimension

$$\varphi(X)=\begin{pmatrix} \text{mean}(X) \\ \text{var}(X) \end{pmatrix}$$

  - Fixed size representation of sets in 2d dimensions
  - Use kernel to compare different sets:

$$k(X_1, X_2)=\langle \varphi(X_1), \varphi(X_2) \rangle$$

# Fisher kernels

- Proposed by Jaakkola & Haussler, "Exploiting generative models in discriminative classifiers",In *Advances in Neural Information Processing Systems* 11, 1998

- Motivated by the need to represent variably sized objects in a vector space, such as sequences, sets, trees, graphs, etc., such that they become amenable to be used with linear classifiers, and other data analysis tools

- A generic method to define kernels over arbitrary data types based on generative statistical models.
  - Define a probability distribution over the items we want to represent

$$p(x;\theta), \quad x \in X, \quad \theta \in R^D$$

# Fisher kernels

- Given a generative data model $p(x;\theta),\ \ x \in X,\ \ \theta \in R^D$
- Represent data x in X by means of the gradient of the data log-likelihood, or "Fisher score":

$$g(x) = \nabla_\theta \ln p(x),$$
$$g(x) \in R^D$$

- Define a kernel over X by taking the scaled inner product between the Fisher score vectors:

$$k(x,y) = g(x)^T F^{-1} g(y)$$

- Where F is the Fisher information matrix F:

$$F = \boldsymbol{E}_{p(x)}\left[g(x)\, g(x)^T\right]$$

- F is positive definite since

$$\alpha^T F \alpha = \boldsymbol{E}_{p(x)}\left[\left(g(x)^T \alpha\right)^2\right] > 0$$

# Fisher kernels

- The Fisher score has zero mean under the generative model

$$\begin{aligned} E_{p(x)}[g(x)] &= \int_x p(x) \frac{\partial}{\partial \theta} \ln p(x) \\ &= \int_x p(x) \frac{1}{p(x)} \frac{\partial}{\partial \theta} p(x) \\ &= \int_x \frac{\partial}{\partial \theta} p(x) \\ &= \frac{\partial}{\partial \theta} \int_x p(x) \\ &= \frac{\partial}{\partial \theta} 1 \\ &= 0 \end{aligned}$$

- Therefore, the Fisher information matrix is the covariance matrix of the Fisher score under the generative model

$$F = \boldsymbol{E}_{p(x)}\left[g(x) g(x)^T\right]$$

# Fisher vector

- Since F is positive definite we can decompose its inverse as

$$F^{-1} = L^T L$$

- Therefore, we can write the kernel as

$$k(x_i, x_j) = g(x_i)^T F^{-1} g(x_j) = \varphi(x_i)^T \varphi(x_j)$$

  - Where phi is known as the **Fisher vector**

$$\varphi(x_i) = L\, g(x_i)$$

- From this explicit finite-dimensional data embedding it follows immediately that the Fisher kernel is a positive-semidefinite

- Since F is covariance of Fisher score, normalization by L makes the Fisher vector have unit covariance matrix under p(x)

# Normalization with inverse Fisher information matrix

- Gradient of log-likelihood w.r.t. parameters $g(x) = \nabla_\theta \ln p(x)$
- Fisher information matrix $F_\theta = \int g(x) g(x)^T p(x) dx$
- Normalized Fisher kernel $k(x_1, x_2) = g(x_1)^T F_\theta^{-1} g(x_2)$
  - Renders Fisher kernel invariant for parametrization

- Consider different parametrization given by some invertible function $\lambda = f(\theta)$
- Jacobian matrix relating the parametrizations $[J]_{ij} = \frac{\partial \theta_j}{\partial \lambda_i}$
- Gradient of log-likelihood w.r.t. new parameters

$$h(x) = \nabla_\lambda \ln p(x) = J \nabla_\theta \ln p(x) = J g(x)$$

- Fisher information matrix $F_\lambda = \int h(x) h(x)^T p(x) dx = J F_\theta J^T$
- Normalized Fisher kernel

$$\begin{aligned} h(x_1)^T F_\lambda^{-1} h(x_2) &= g(x_1)^T J^T (J F_\theta J^T)^{-1} J g(x_2) \\ &= g(x_1)^T J^T J^{-T} F_\theta^{-1} J^{-1} J g(x_2) \\ &= g(x_1)^T F_\theta^{-1} g(x_2) \\ &= k(x_1, x_2) \end{aligned}$$

# Data-adaptive kernel design

- Fisher vector given by linear projection of gradient

$$\varphi(x) = F^{-1/2} \nabla_\theta \ln p(x)$$

- Fisher kernel is dot-product over Fisher vectors

$$k(x_i, x_j) = \varphi(x_i)^T \varphi(x_j)$$

- Parameters of the model p(x) estimated from data
  - Structure typically determined manually in advance
- Data characteristics captured by Fisher vector depend on data used to train the generative model
- Semi-automatic data-driven kernel design instead of predominant completely manual design

# Fisher kernels: example with Gaussian data model

- Let lambda be the inverse variance, i.e. precision, parameter

$$p(x) = N(x; \mu, \lambda) = \sqrt{\lambda/(2\pi)} \exp\left[-\frac{1}{2}\lambda(x-\mu)^2\right]$$

$$\ln p(x) = \frac{1}{2}\ln\lambda - \frac{1}{2}\ln(2\pi) - \frac{1}{2}\lambda(x-\mu)^2$$

$$\theta = (\mu, \lambda)^T$$

- The partial derivatives are found to be

$$\frac{\partial \ln p(x)}{\partial \mu} = \lambda(x-\mu) \qquad \frac{\partial \ln p(x)}{\partial \lambda} = \frac{1}{2}\left[\lambda^{-1} - (x-\mu)^2\right]$$

# Fisher kernels: example with Gaussian data model

- The partial derivatives are found to be

$$\frac{\partial \ln p(x)}{\partial \mu} = \lambda (x-\mu) \qquad \frac{\partial \ln p(x)}{\partial \lambda} = \frac{1}{2}\left[\lambda^{-1} - (x-\mu)^2\right]$$

- Using central 3rd and 4th Gaussian moment, we get Fisher Information matrix

$$E_p[(x-\mu)^3] = 0$$

$$E_p[(x-\mu)^4] = 3\sigma^4$$

$$F = \begin{pmatrix} \lambda & 0 \\ 0 & \frac{1}{2}\lambda^{-2} \end{pmatrix}$$

- The Fisher vector is then

$$\varphi(x) = \begin{pmatrix} (x-\mu)/\sigma \\ \frac{1}{\sqrt{2}}\left(1-(x-\mu)^2/\sigma^2\right) \end{pmatrix}$$

# Fisher kernels: example with Gaussian data model

- Now suppose an i.i.d. data model over a set of data points

$$p(x) = N(x;\mu,\lambda) = \sqrt{\lambda/(2\pi)}\exp\left[-\frac{1}{2}\lambda(x-\mu)^2\right]$$

$$p(X) = p(x_{1,\dots,}x_N) = \prod_{i=1}^{N} p(x_i)$$

- Then the Fisher vector is given by the sum of Fisher vectors of the points
  - Encodes the discrepancy in the first and second order moment of the data w.r.t. those of the model

$$\varphi(X) = \sum_{i=1}^{N} \varphi(x_i) = N\begin{pmatrix} (\hat{\mu}-\mu)/\sigma \\ (\sigma^2-\hat{\sigma}^2)/(\sigma^2\sqrt{2}) \end{pmatrix}$$

  - Where

$$\hat{\mu} = \frac{1}{N}\sum_{i=1}^{N} x_i \qquad \hat{\sigma}^2 = \frac{1}{N}\sum_{i=1}^{N} (x_i-\mu)^2$$

# Fisher kernels – relation to generative classification

- Suppose we make use of generative model for classification via Bayes' rule
  - Where x is the data to be classified, and y is the discrete class label

$$p(y|x) = p(x|y)\,p(y)/p(x),$$
$$p(x) = \sum_{k=1}^{K} p(y=k)\,p(x|y=k)$$

and

$$p(x|y) = p(x;\theta_y),$$
$$p(y=k) = \pi_k = \frac{\exp(\alpha_k)}{\sum_{k'=1}^{K} \exp(\alpha_{k'})}$$

- Classification with the Fisher kernel obtained using the marginal distribution p(x) is at least as powerful as classification with Bayes' rule.
- This becomes useful when the class conditional models are poorly estimated, either due to bias or variance type of errors.
- In practice often used without class-conditional models, but direct generative model for the marginal distribution on X.

# Fisher kernels – relation to generative classification

- Consider the Fisher score vector with respect to the marginal distribution on X

$$\nabla_\theta \ln p(x) = \frac{1}{p(x)} \nabla_\theta \sum_{k=1}^{K} p(x, y=k)$$
$$= \frac{1}{p(x)} \sum_{k=1}^{K} p(x, y=k) \nabla_\theta \ln p(x, y=k)$$
$$= \sum_{k=1}^{K} p(y=k|x) \left[ \nabla_\theta \ln p(y=k) + \nabla_\theta \ln p(x|y=k) \right]$$

- In particular for the alpha that model the class prior probabilities we have

$$\frac{\partial \ln p(x)}{\partial \alpha_k} = p(y=k|x) - \pi_k$$

# Fisher kernels – relation to generative classification

- First K elements in Fisher score given by class posteriors minus a constant

$$g(x) = \nabla_\theta \ln p(x) = \left( p(y=1|x) - \pi_1, \ldots, p(y=K|x) - \pi_K, \ldots \right)$$

- Consider discriminative multi-class classifier, for the k-th class
  - Let the weight vector be zero, except for the k-th position where it is one
  - Let the bias term be equal to the prior probability of that class
- Then

$$f_k(x) = w_k^T g(x) + b_k = p(y=k|x)$$

and thus

$$\operatorname{argmax}_k \; f_k(x) = \operatorname{argmax}_k \; p(y=k|x)$$

- Thus the Fisher kernel based classifier can implement classification via Bayes' rule, and generalizes it to other classification functions.

# Challenging factors in object recognition

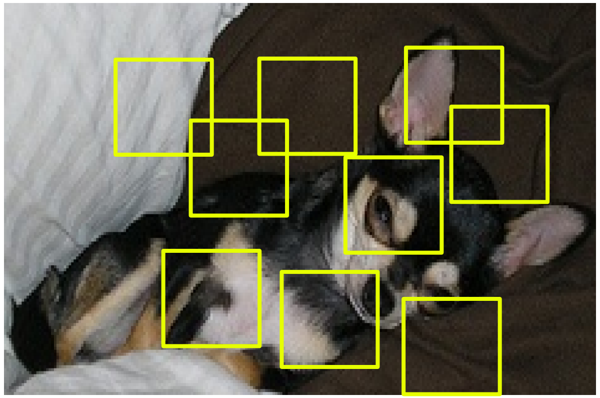
- Intra-class appearance variation
  - Objects deformation due to pose
  - Transparency: e.g. bottles
  - Sub-categories: boat = ferry + yacht +...

- Scene composition
  - Heavy occlusions: e.g. tables and chairs
  - Clutter: coincidental image content present

- Imaging conditions
  - viewpoint, scale, illumination

# Representing images as “bags of features”

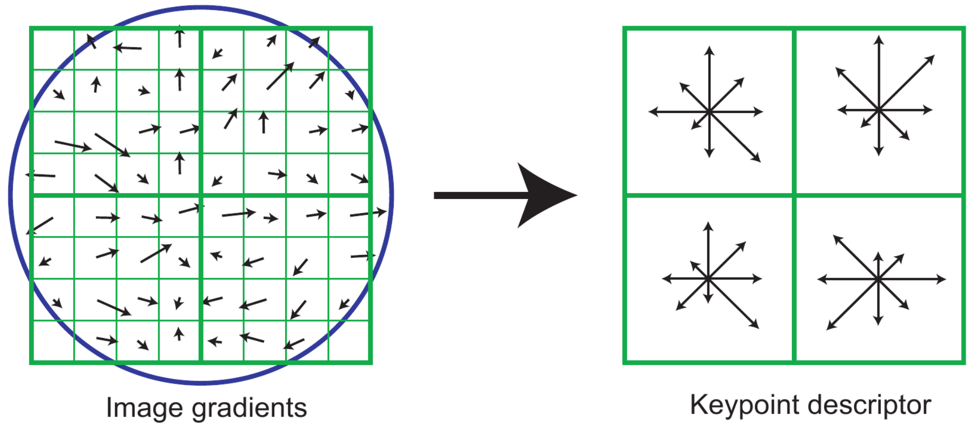
- Global rigid representation likely to be affected by nuisance factors such as deformation, (self-)occlusion, clutter, etc.
- Instead consider local image regions, or “patches”, on which some representation is computed that is (partially) invariant to imaging conditions such as viewpoint, illumination, scale, etc.
  - Local patterns more likely to be preserved, or at least some of them
- Patch extraction and description stage
  - Patch sampling from image on dense multi-scale grid, or interest points
  - Descriptor computation: SIFT, HOG, LBP, color names, …
- Set of local descriptors characterizes the image (or video, or speech, or ...)
- Feature aggregation stage
  - Global image signature computed
  - Can be classified or used for matching
- See e.g. Schmid & Mohr, PAMI, 1997.



# Local descriptor based image representations

- SIFT patch description most popular
  - 4x4 spatial grid
  - 8 bin orientation histogram
  - Lowe, IJCV, 2004

$$X = \{x_{1}, \ldots, x_{N}\}$$

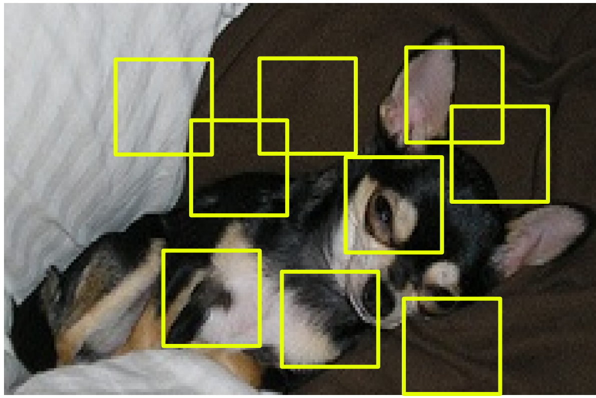



- Coding stage: embed local descriptors, typically in higher dimensional space
  - For example: assignment to cluster indices

$$\varphi(x_i)$$

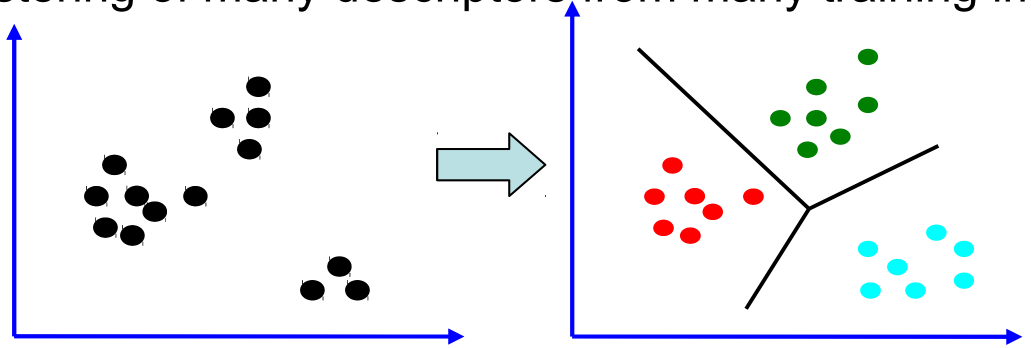
- Pooling stage: aggregate per-patch embeddings
  - For example: sum pooling

$$\Phi(X) = \sum_{i=1}^{N} \varphi(x_i)$$



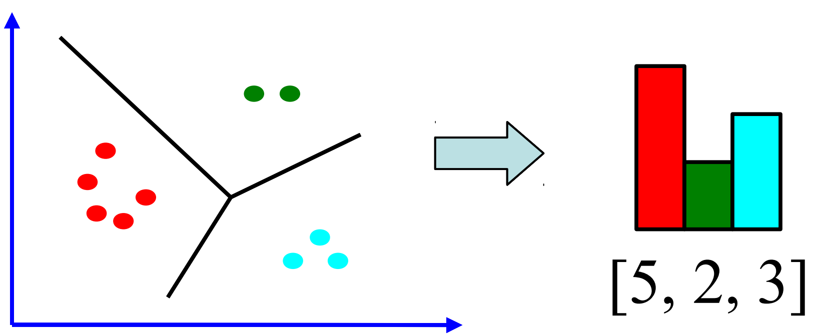
# The “bag of visual words” representation

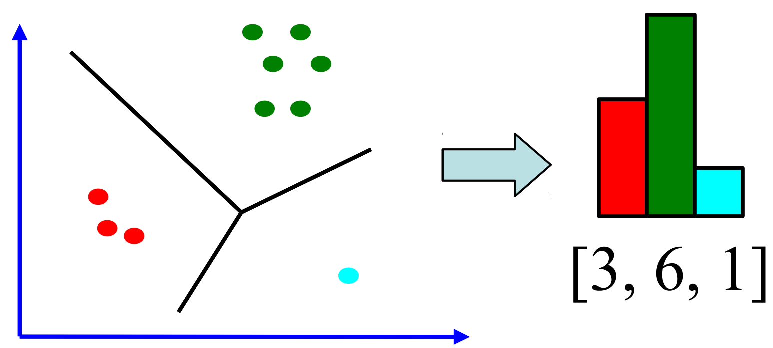
- Offline clustering of many descriptors from many training images



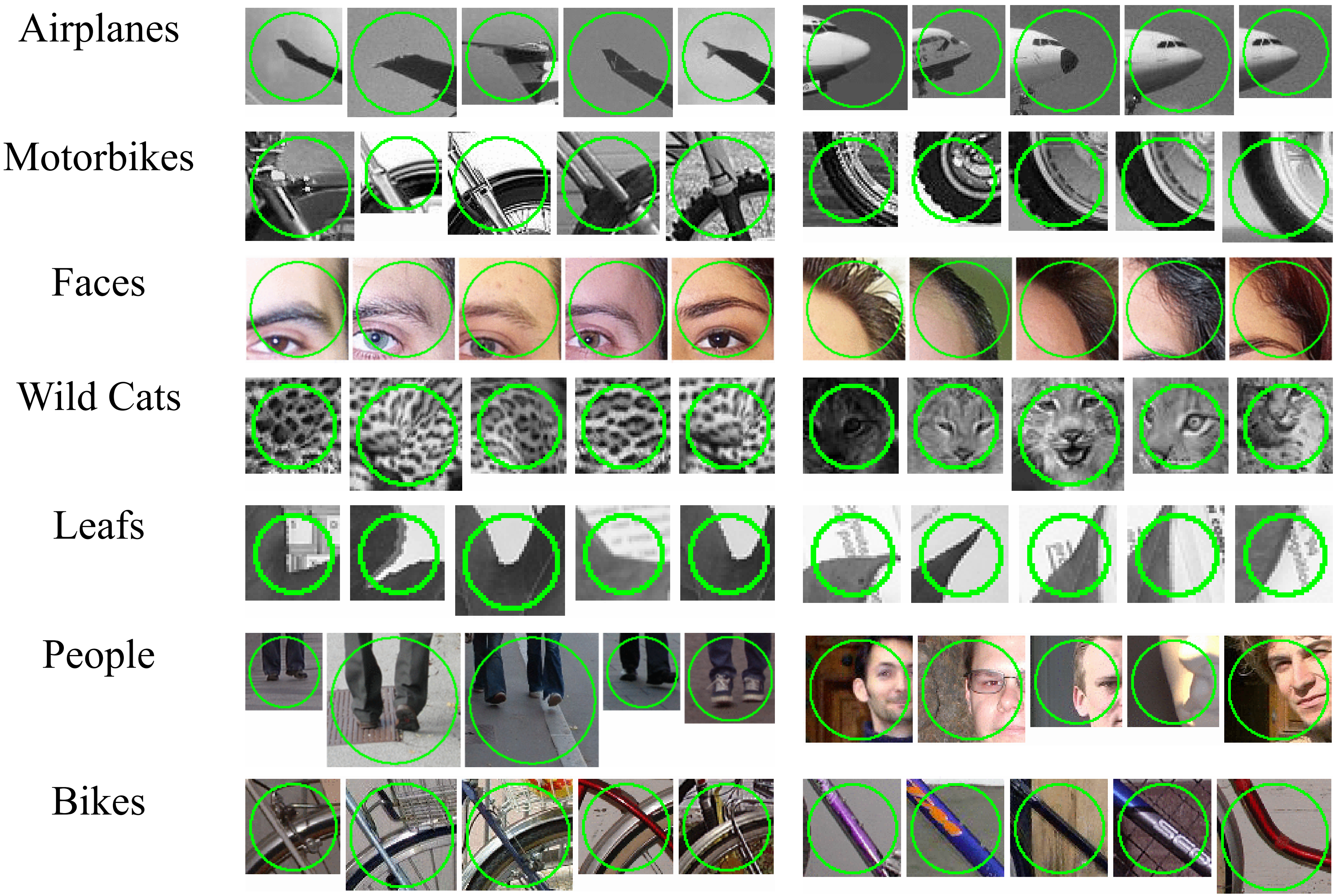
- Encoding a new image:
  - Compute local descriptors, assign to cluster
  - Count histogram of descriptors in each cluster
- Sum pooling of “1-hot encoding” of local descriptors

$$\varphi(x_i) = [0, \ldots, 0, 1, 0, \ldots, 0] \qquad h = \sum_i \varphi(x_i)$$



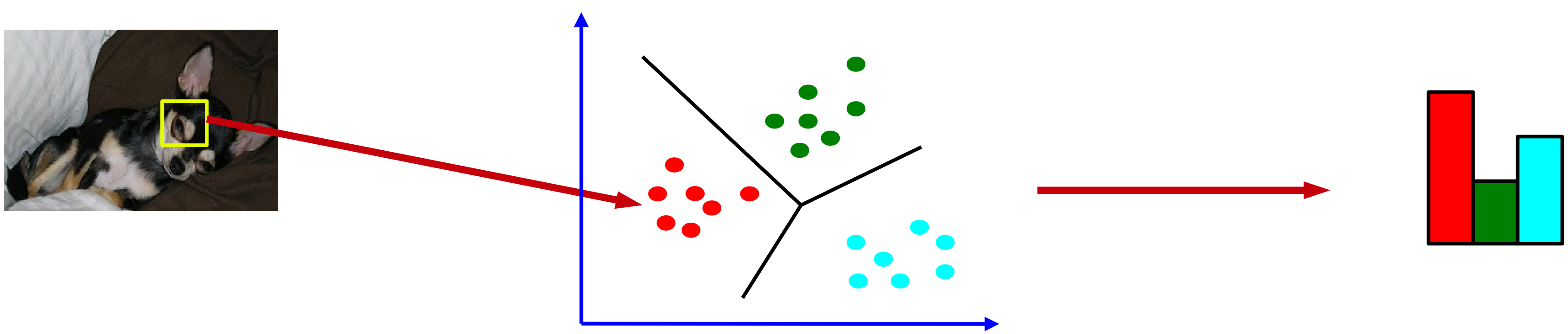



# Example visual words found by clustering



# Application of FV for bag-of-words image-representation

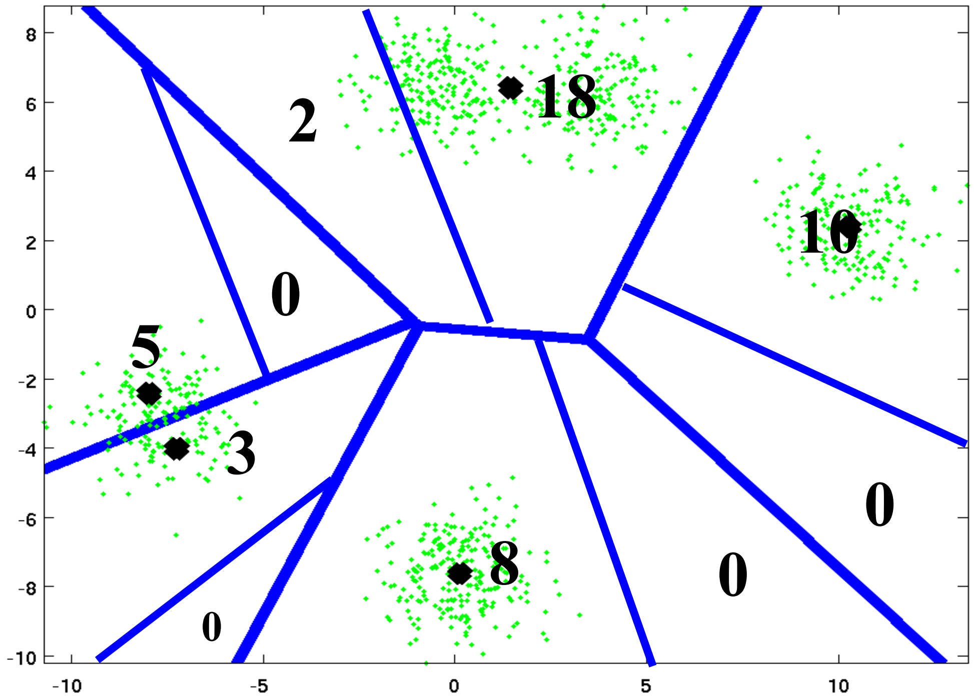
- Bag of word (BoW) representation
  - Map every descriptor to a cluster / visual word index $w_i \in \{1, \dots, K\}$
- Model visual word indices with i.i.d. multinomial $p(w_i = k) = \frac{\exp \alpha_k}{\sum_{k'} \exp \alpha_{k'}} = \pi_k$
  - Likelihood of N i.i.d. indices: $p(w_{1:N}) = \prod_{i=1}^{N} p(w_i)$
  - Fisher vector given by gradient $\frac{\partial \ln p(w_{1:N})}{\partial \alpha_k} = \sum_{i=1}^{N} \frac{\partial \ln p(w_i)}{\partial \alpha_k} = h_k - N \pi_k$
    - i.e. BoW histogram + constant



# Fisher vector GMM representation: Motivation

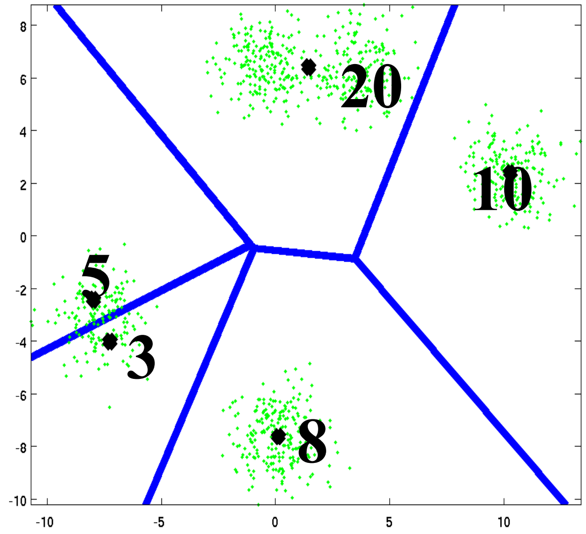
- Suppose we want to refine a given visual vocabulary to obtain a richer image representation

- Bag-of-word histogram stores # patches assigned to each word
  - Need more words to refine the representation
  - But this directly increases the computational cost
  - And leads to many empty bins: redundancy



# Fisher vector GMM representation: Motivation

- Feature vector quantization is computationally expensive
- To extract visual word histogram for a new image
  - Compute distance of each local descriptor to each k-means center
  - run-time O(NKD) : linear in
    - N: nr. of feature vectors ~ $10^4$ per image
    - K: nr. of clusters ~ $10^3$ for recognition
    - D: nr. of dimensions ~ $10^2$ (SIFT)



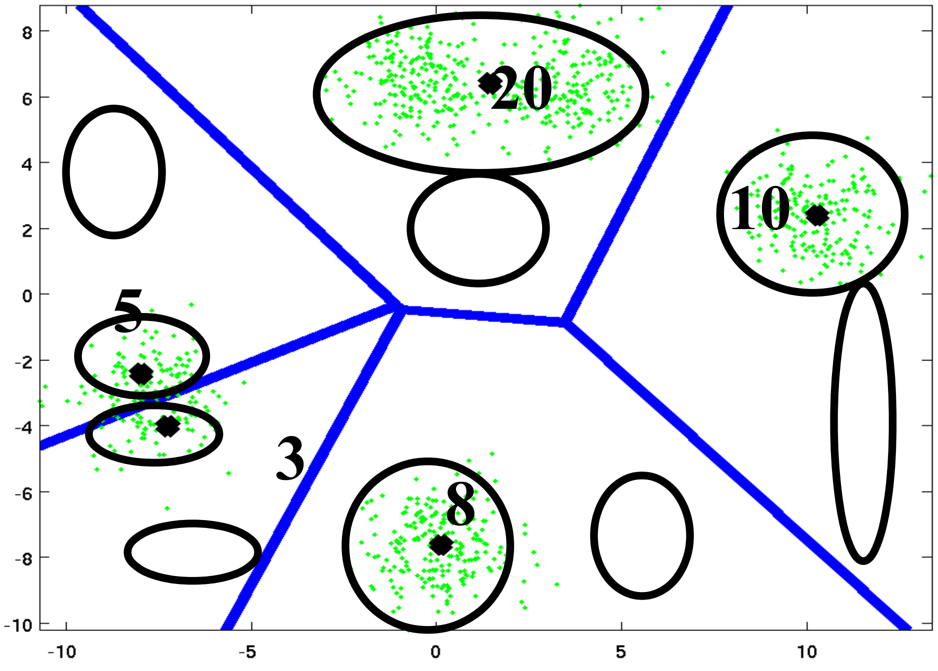

- So in total in the order of $10^9$ multiplications per image to obtain a histogram of size 1000

- Can this be done more efficiently ?!
  - Yes, extract more than just a visual word histogram from a given clustering

# Fisher vector representation in a nutshell

- Instead, the Fisher Vector for GMM also records the mean and variance of the points per dimension in each cell
  - More information for same # visual words
  - Does not increase computational time significantly
  - Leads to high-dimensional feature vectors
- Even when the counts are the same,

  the position and variance of the points in the cell can vary



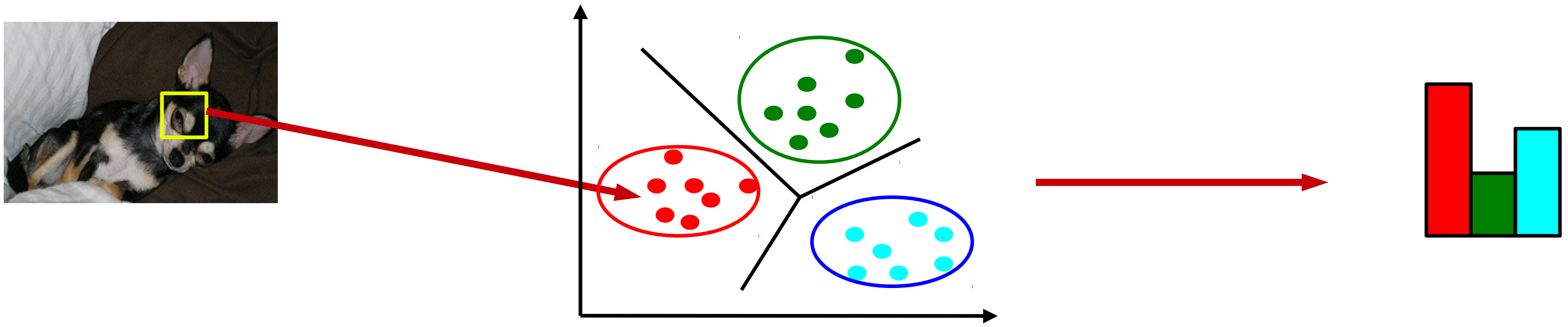
# Application of FV for Gaussian mixture model of local features

- Gaussian mixture models for local image descriptors
  **[Perronnin & Dance, CVPR 2007]**
  - State-of-the-art feature pooling for image/video classification/retrieval

- Offline: Train k-component GMM on collection of local features

$$p(x) = \sum_{k=1}^{K} \pi_k N(x; \mu_k, \sigma_k)$$

- Each mixture component corresponds to a visual word
  - Parameters of each component: mean, variance, mixing weight
  - We use diagonal covariance matrix for simplicity
    - Coordinates assumed independent, locally per Gaussian



# Application of FV for Gaussian mixture model of local features

- Model local image features with Gaussian mixture model

$$p(x)=\sum_{k=1}^{K}\pi_k N(x;\mu_k,\sigma_k)$$

- Fisher vector representation: gradient of log-likelihood
  - For the means and variances we have:

$$F^{-1/2}\nabla_{\mu_k}\ln p(x_{1:N})=\frac{1}{\sqrt{\pi_k}}\sum_{n=1}^{N}p(k|x_n)\frac{(x_n-\mu_k)}{\sigma_k}$$

$$F^{-1/2}\nabla_{\sigma_k}\ln p(x_{1:N})=\frac{1}{\sqrt{2\pi_k}}\sum_{n=1}^{N}p(k|x_n)\left\{\frac{(x_n-\mu_k)^2}{\sigma_k^2}-1\right\}$$

  - Soft-assignments given by component posteriors

$$p(k|x_n)=\frac{\pi_k N(x_n;\mu_k,\sigma_k)}{p(x_n)}$$

# Application of FV for Gaussian mixture model of local features

- Fisher vector components give the difference between the data mean predicted by the model and observed in the data, and similar for variance.

- For the gradient w.r.t. the mean

$$F^{-1/2}\nabla_{\mu_k} \ln p(x_{1:N}) = \frac{1}{\sqrt{\pi_k}} \sum_{n=1}^{N} p(k|x_n) \frac{(x_n - \mu_k)}{\sigma_k} = \frac{n_k}{\sigma_k \sqrt{\pi_k}} (\hat{\mu}_k - \mu_k)$$

  - where $n_k = \sum_{n=1}^{N} p(k|x_n)$ $\hat{\mu}_k = n_k^{-1} \sum_{n=1}^{N} p(k|x_n) x_n$

- Similar for the gradient w.r.t. the variance

$$F^{-1/2}\nabla_{\sigma_k} \ln p(x_{1:N}) = \frac{1}{\sqrt{2\pi_k}} \sum_{n=1}^{N} p(k|x_n) \left\{ \frac{(x_n - \mu_k)^2}{\sigma_k^2} - 1 \right\} = \frac{n_k}{\sigma_k^2 \sqrt{2\pi_k}} (\hat{\sigma}_k^2 - \sigma_k^2)$$

  - where $\hat{\sigma}_k^2 = n_k^{-1} \sum_{n=1}^{N} p(k|x_n)(x_n - \mu_k)^2$

# Image representation using Fisher kernels

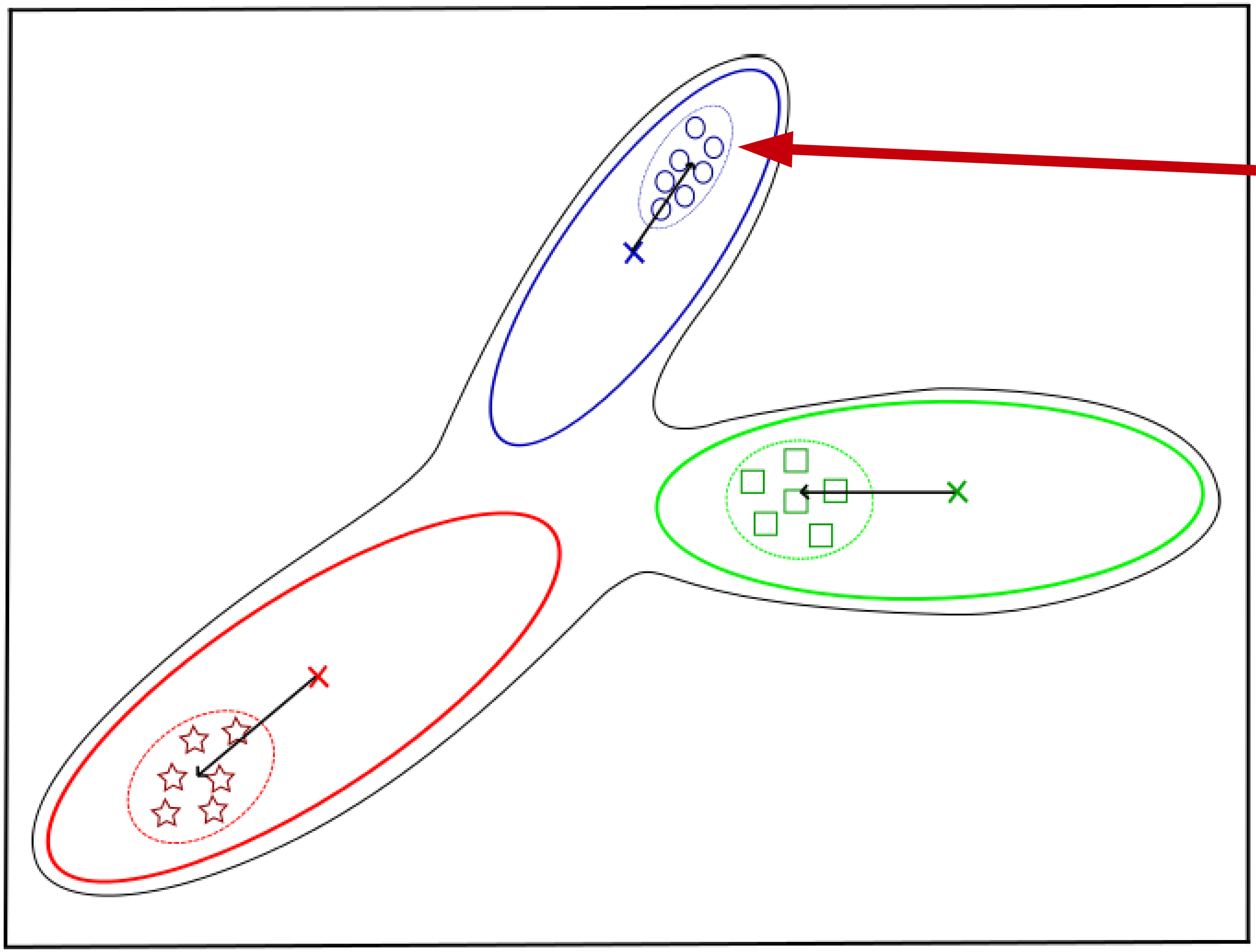
- Data representation

$$G(X,\Theta) = F^{-1/2} \left( \frac{\partial L}{\partial \alpha_1}, \ \dots \ , \frac{\partial L}{\partial \alpha_K} \ , \ \nabla_{\mu_1} L, \ \dots \ , \nabla_{\mu_K} L \ , \ \nabla_{\sigma_1} L, \ \dots \ , \ \nabla_{\sigma_K} L \right)^T$$

- In total K(1+2D) dimensional representation, since for each visual word / Gaussian we have
  - Mixing weight (1 scalar)
  - Mean (D dimensions)
  - Variances (D dimensions, since single variance per dimension)

- Gradient with respect to mixing weights often dropped in practice since it adds little discriminative information for classification.
  - Results in 2KD dimensional image descriptor

# Illustration of gradient w.r.t. means of Gaussians



New Data Points

# BoW and FV from a function approximation viewpoint

- Let us consider uni-dimensional descriptors: vocabulary quantizes real line
- For both BoW and FV the representation of an image is obtained by sum-pooling the representations of descriptors.
  - Ensemble of descriptors sampled in an image $X=\{x_{1,},\ldots,x_N\}$
  - Representation of single descriptor
    - One-of-k encoding for BoW $\varphi(x_i)=[0,\ldots,0,1,0,\ldots,0]$
    - For FV concatenate per-visual word gradients of form

$$\varphi(x_i)=\left(\ldots,p(k|x_i)\left[1 \quad \frac{(x_i-\mu_k)}{\sigma_k} \quad \frac{(x_i-\mu_k)^2-\sigma_k^2}{\sigma_k^2}\right],\ldots\right)$$

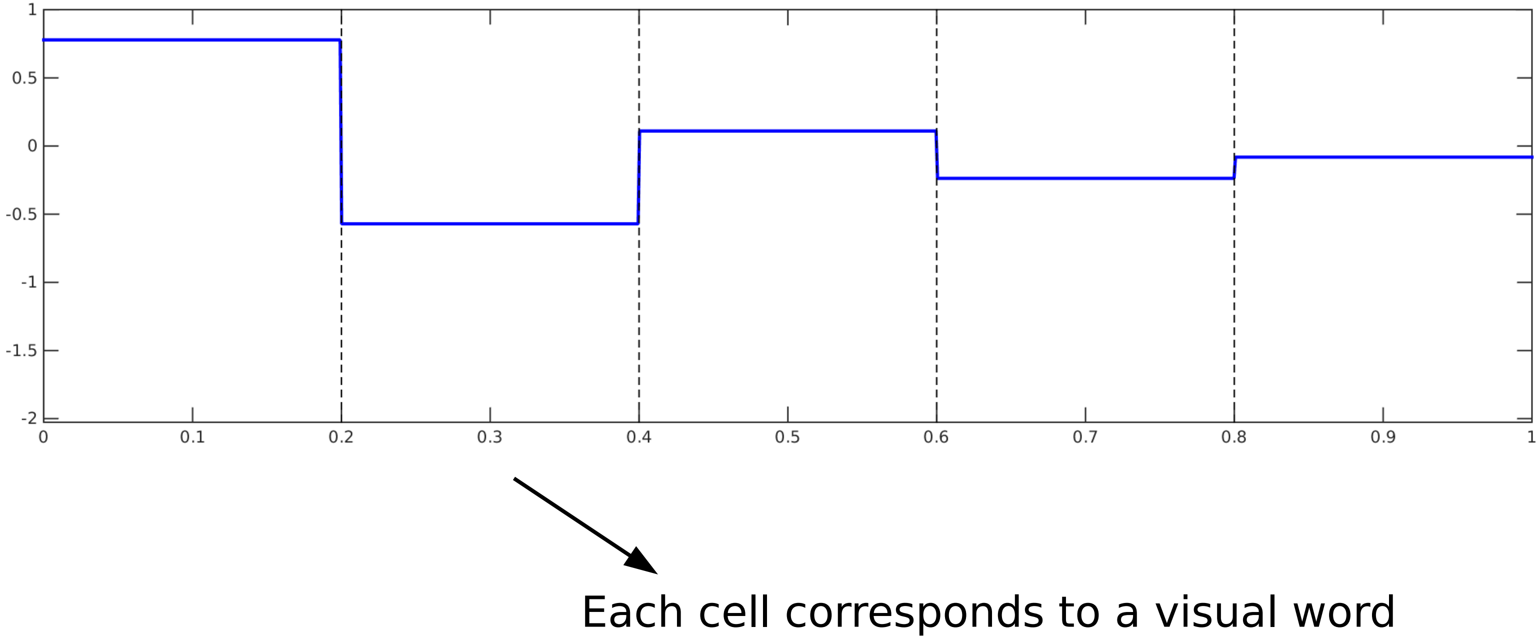
- Linear function of sum-pooled descriptor encodings is a sum of linear functions of individual descriptor encodings:

$$\Phi(X)=\sum_{i=1}^{N}\varphi(x_i)$$

$$w^T\Phi(X)=\sum_{i=1}^{N}w^T\varphi(x_i)$$

# From a function approximation viewpoint

- Consider the score of a single descriptor for BoW
  - If assigned to k-th visual word then $w^T \varphi(x_i) = w_k$
  - Thus: constant score for all descriptors assigned to a visual word

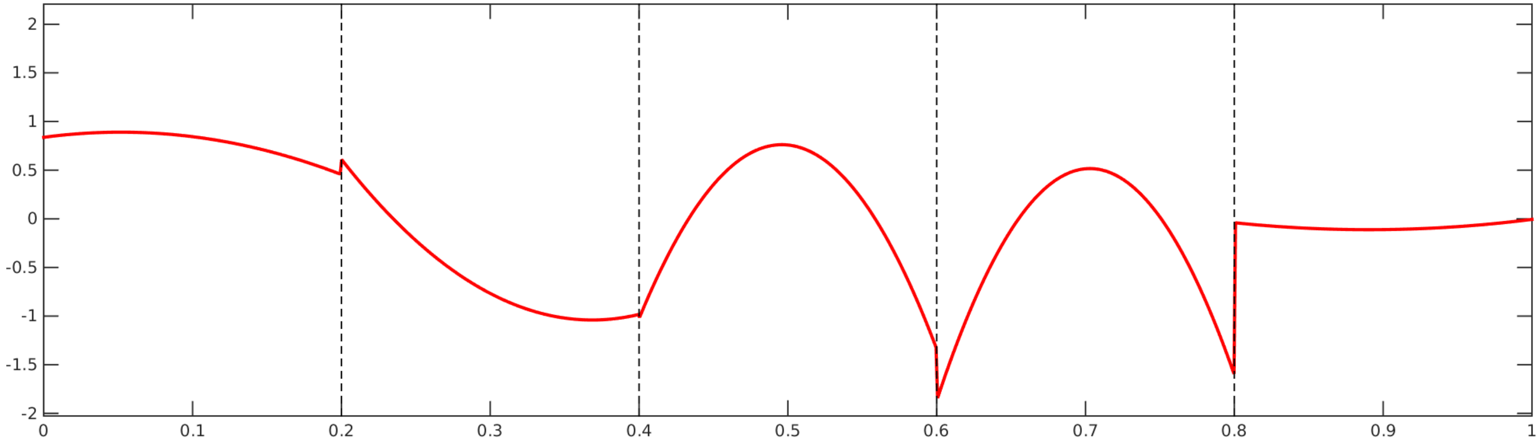


## From a function approximation viewpoint

- Consider the same for FV, and assume soft-assignment is “hard”
  - Thus: assume for one value of k we have $p(k|x_i) \approx 1$
  - If assigned to the k-th visual word:

$$w^T \varphi(x_i) = w_k^T \begin{bmatrix} 1 & \frac{(x_i - \mu_k)}{\sigma_k} & \frac{(x_i - \mu_k)^2 - \sigma_k^2}{\sigma_k^2} \end{bmatrix}$$

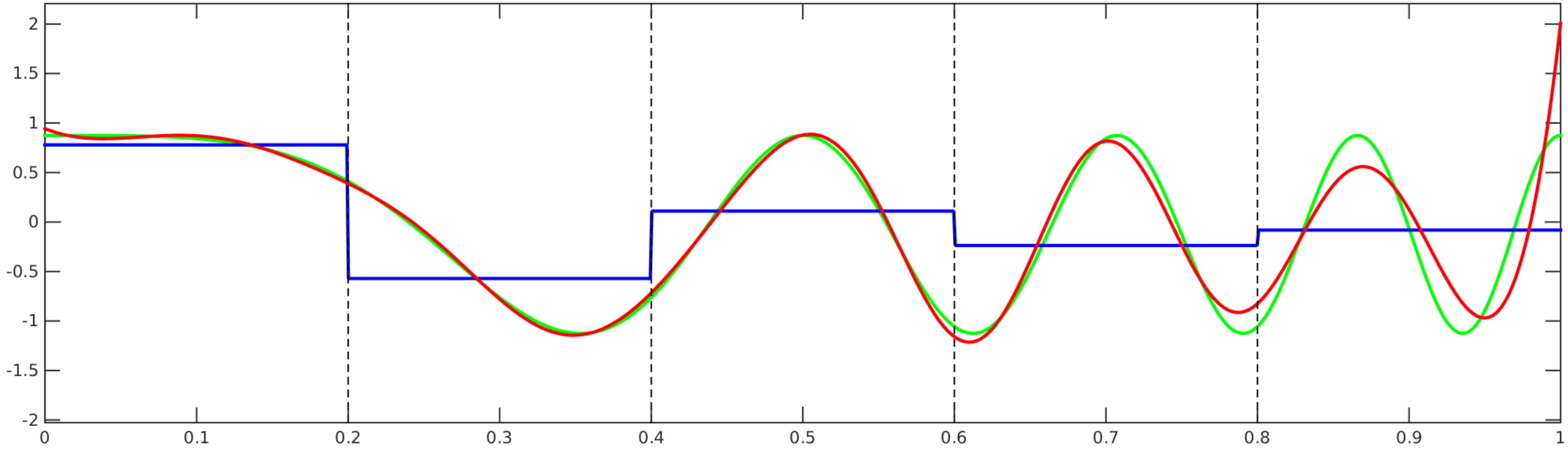
    - Note that $w_k$ is no longer a scalar but a vector
  - Thus: score is a second-order polynomial of the descriptor x, for descriptors assigned to a given visual word.



# From a function approximation viewpoint

- Consider that we want to approximate a true classification function (green) based on either BoW (blue) or FV (red) representation
  - Weights for BoW and FV representation fitted by least squares to optimally match the target function

- Better approximation with FV
  - Local second order approximation, instead of local zero-order
  - Smooth transition from one visual word to the next



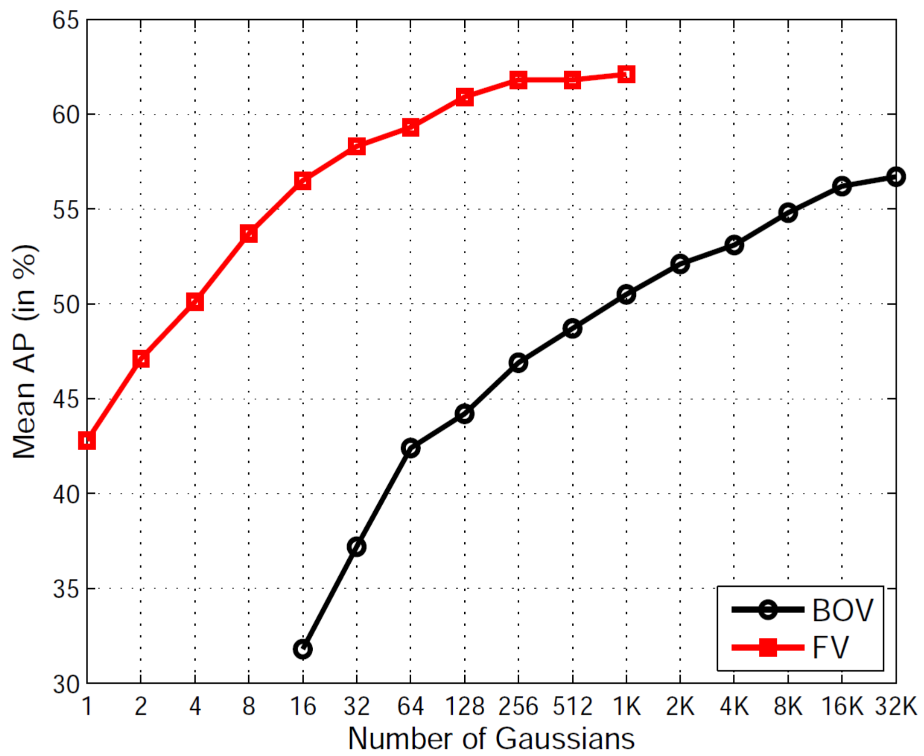
# Fisher vectors: classification performance VOC'07

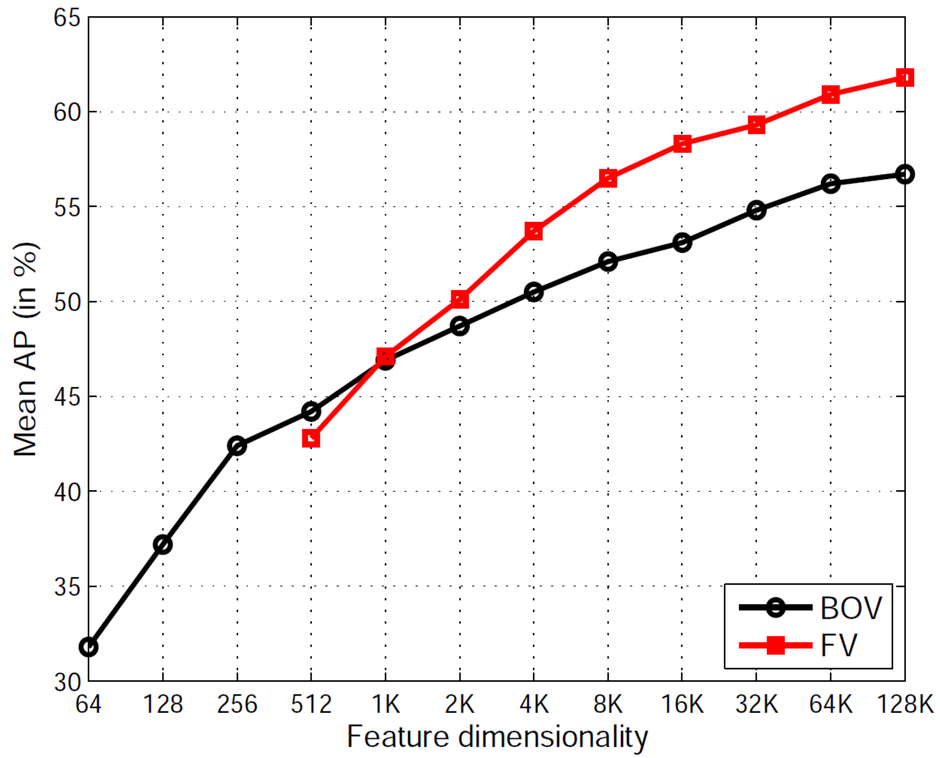
- Yearly evaluation from 2005 to 2012 for image classification

# Fisher vectors: classification performance VOC'07

- Fisher vector representation yields better performance for a given number of Gaussians / visual words than Bag-of-words.

- For a fixed dimensionality Fisher vectors perform better, and are more efficient to compute



# Normalization of the Fisher vector

- Inverse Fisher information matrix *F*
  - Renders FV invariant for re-parametrization
  - Linear projection, analytical approximation for MoG gives diagonal matrix
    [Sanchez, Perronnin, Mensink, Verbeek IJCV'13]

$$F = E\left[g(x)\, g(x)^T\right]$$

$$f(x) = F^{-1/2} g(x)$$

- Power-normalization, applied independently per dimension
  - Renders Fisher vector less sparse
    [Perronnin, Sanchez, Mensink, ECCV'10]
  - Corrects for poor independence assumption on local descriptors
    [Cinbis, Verbeek, Schmid, PAMI'15]

$$f(x) \leftarrow sign(f(x))\,|f(x)|^{\rho}$$

$$0 < \rho < 1$$

- L2-normalization
  - Makes representation invariant to number of local features
  - Among other Lp norms the most effective with linear classifier
    [Sanchez, Perronnin, Mensink, Verbeek IJCV'13]

$$f(x) \leftarrow \frac{f(x)}{\sqrt{f(x)^T f(x)}}$$

# Effect of power and L2 normalization in practice

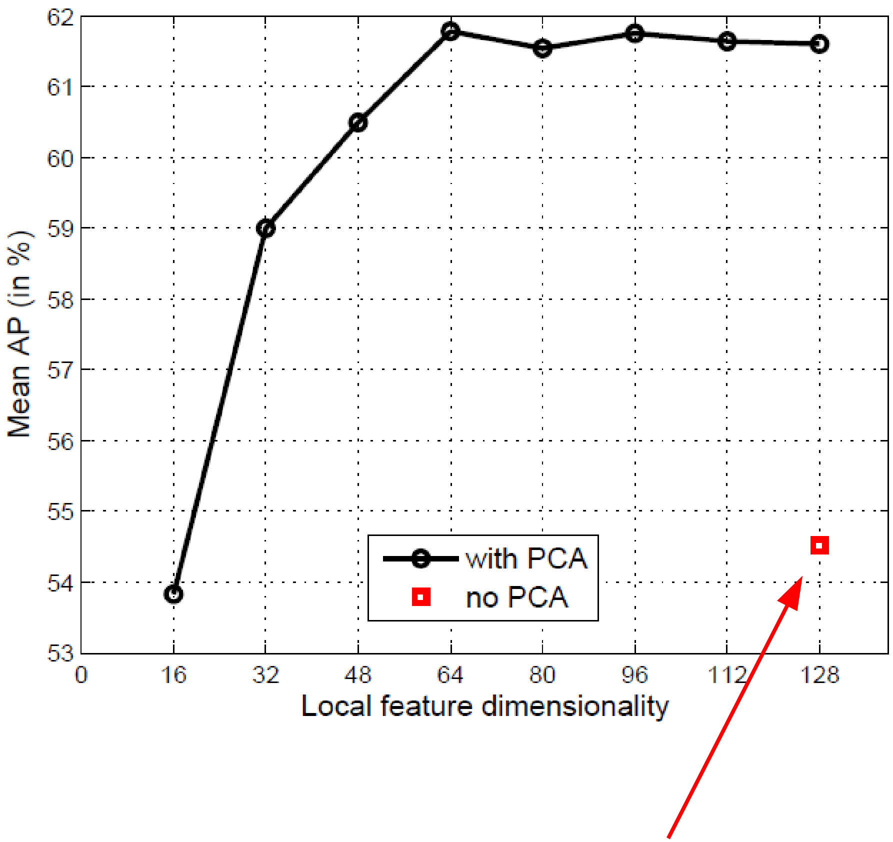
- Classification results on the PASCAL VOC 2007 benchmark dataset.
- Regular dense sampling of local SIFT descriptors in the image
  - PCA projected to 64 dimensions to de-correlate and compress
- Using mixture of 256 Gaussians over the SIFT descriptors
  - FV dimensionality: 2*64*256 = 32 * 1024

| Power Nomalization | L2 normalization | Performance (mAP) | Improvement over baseline |
|---|---|---|---|
| No | No | 51.5 | 0 |
| Yes | No | 59.8 | 8.3 |
| No | Yes | 57.3 | 5.8 |
| Yes | Yes | 61.8 | 10.3 |

# PCA dimension reduction of local descriptors

- We use diagonal covariance model
- Dimensions might be correlated
- Apply PCA projection to
  - De-correlate features
  - Reduce dimension of final FV
- FV with 256 Gaussians over local SIFT descriptors of dimension 128

Results on PASCAL VOC'07:



# Example applications: Fine-grained classification

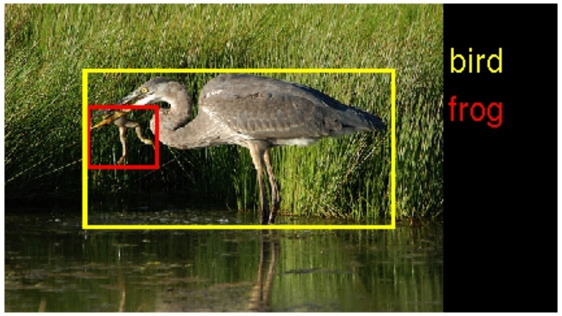
aircraft (100) birds (83) cars (196) dogs (120) shoes (70)

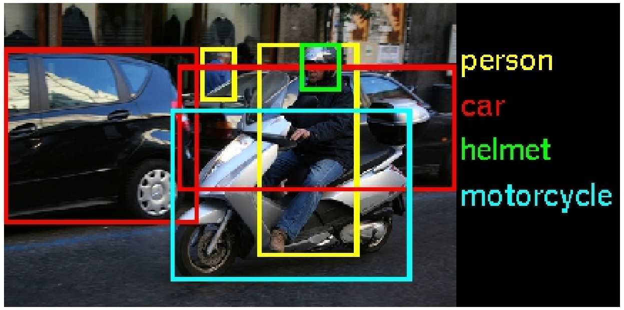
- Winning INRIA+Xerox system at FGComp'13: http://sites.google.com/site/fgcomp2013
  - multiple low-level descriptors: SIFT, color, etc.
  - Fisher Vector embedding

    [Gosselin, Murray, Jégou, Perronnin, "Revisiting the Fisher vector for fine-grained classification", PRL'14.]

- Many other successful uses of FVs for fine-grained recognition
  - Rodriguez and Larlus, "Predicting an object location using a global image representation", ICCV'13.
  - Gavves, Fernando, Snoek, Smeulders, Tuytelaars, "Fine-Grained Categorization by Alignments", ICCV'13
  - Murray, Perronnin, "Generalized Max Pooling", CVPR'14.

# Example applications: object detection


- ImageNet'13 detection: http://www.image-net.org/challenges/LSVRC/2013/
- Winning system by University of Amsterdam
  - region proposals with selective search
  - Fisher Vector embedding
  - Fast Local Area Independent Representation (FLAIR)

**Van de Sande, Snoek, Smeulders, "Fisher and VLAD with FLAIR", CVPR'14.**

# Example applications: face verification

- Face track description:
  - track face
  - extract SIFT descriptors
  - encode using Fisher vectors
  - pool at face track level

**Parkhi, Simonyan, Veldaldi, Zisserman, "A compact and discriminative face track descriptor", CVPR'14.**

- New state-of-the-art results on the YouTube faces dataset

| | Method | Accuracy | AUC | EER |
|---|---|---|---|---|
| 1 | MGBS & SVM- [37] | $78.9 \pm 1.9$ | 86.9 | 21.2 |
| 2 | APEM FUSION [20] | $79.1 \pm 1.5$ | 86.6 | 21.4 |
| 3 | STFRD & PMML [11] | $79.5 \pm 2.5$ | 88.6 | 19.9 |
| 4 | VSOF & OSS (Adaboost) [22] | $79.7 \pm 1.8$ | 89.4 | 20.0 |
| 5 | Our $VF^2$ (restricted) | $83.5 \pm 2.3$ | 92.0 | 16.1 |
| 6 | Our $VF^2$ (restricted & flip) | $84.7 \pm 1.4$ | 93.0 | 14.9 |
| 7 | Our $VF^2$ (unrestricted & flip) | $83.5 \pm 2.1$ | 94.0 | 13.0 |
| 8 | Our $VF^2$ (unrestricted & jitt. pool.) | $83.8 \pm 1.6$ | 95.0 | **12.3** |

# Example: action recognition and localization


- THUMOS action recognition challenge 2013 & 2014

  http://crcv.ucf.edu/ICCV13-Action-Workshop

- Winning systems by INRIA-LEAR
  - improved dense trajectory video features
  - Fisher Vector embedding

**Wang, Oneata, Verbeek and Schmid, "A robust and efficient video representation for action recognition", IJCV'15.**

# Bag-of-words vs. Fisher vector representation

- Bag-of-words image representation
  - k-means clustering
  - histogram of visual word counts, K dimensions
- Fisher vector image representation
  - GMM clustering
  - Local first and second order moments, 2KD dimensions
- For a given dimension of the representation
  - FV needs less clusters, and is faster to compute
  - FV gives better performance since it is a smoother function of the local descriptors.
- Power and L2 normalizations effective for both representations